Journal of Engineering Physics and Thermophysics, Vol. 80, No. 2, 2007

CONCERNING THE FILTRATIONAL COOLING OF A HEAT-RELEASING GRANULAR BED IN THE PRESENCE OF A FIRST-ORDER PHASE TRANSITION

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UDC 532.546

Within the framework of a two-temperature approximation, mathematical simulation of the process of filtrational evaporative cooling of a heat-releasing bed is performed. Based on this, an engineering method for calculating the cooling of a granular bed has been developed; it allows one to determine the position and size of the evaporation zone and pressure drops in the system. The conditions for the existence of various regimes of evaporative cooling (the presence of one, two, or three zones of heating and evaporation of a heat carrier) have been obtained in dimensionless form.

Introduction. As is known [1], liquid evaporative cooling of a heat-releasing disperse bed possesses a number of qualitatively new properties as compared to similar processes, the heat carrier in which is a gas. The major features of such a process are: the high heat-transfer intensity in phase conversion of a heat carrier inside a granular bed, a substantial increase in the efficiency of cooling due to the heat of vaporization, and a low flow rate of a liquid coolant.

Despite the fact that the major principles of this method of cooling have been known for a long time [2], up to now there has not been a single consistent method of calculating heat exchangers with bulk heat generation in the presence of phase transition [3]. We note that most of the works published are devoted to theoretical and experimental studies of liquid evaporation in a porous heat-releasing element in application to cooling of the thermally stressed elements of the structures of flying vehicles [3]. A granular bed predominantly consisting of particles of regular shape differs substantially in hydraulic and thermophysical properties from a porous matrix with a very small specific volume, which leads to essential differences in the processes of heat transfer in these systems. The information on heat-releasing disperse media is fragmentary (see, e.g., [4–7]) and does not allow one to reliably calculate the processes proceeding during the flow and evaporation of a liquid heat carrier with allowance for geometric and transfer characteristics of the bed.

In the present work, the task was set to develop a simple engineering method for calculating evaporative cooling of a heat-releasing granular bed for a wide range of acting conditions on the basis of mathematical simulation that takes into account basic thermohydraulic features of the process.

Basic Assumptions. A simplified scheme of evaporative cooling of a heat-releasing bed is presented in Fig. 1. The disperse bed consists of spherical particles of diameter d. A liquid of flow rate J_f and temperature T_0 is supplied to the bed inlet. In the general case, there are three zones: I, the zone of liquid motion, II, the zone of evaporation, and III, the zone of vapor motion. In formulating the model, the following assumptions were made:

1) there are two interpenetrating continua (a heat carrier — solid particles);
2) the process is stationary, the position of the evaporation zone does not change in time;
3) the power of heat release is constant;
4) vapor is considered as a perfect gas;
5) the slip of the heat-carrier phases in the evaporation zone is neglected;

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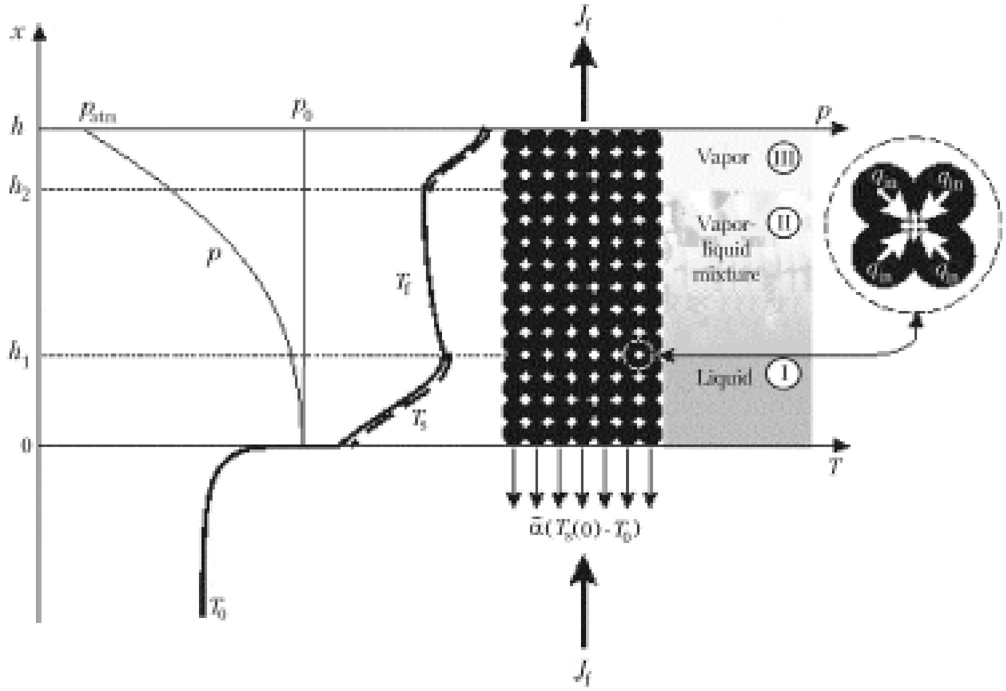

Fig. 1. Schematic diagram of evaporative cooling of a heat-releasing granular bed.

6) the values of dT_f/dx and dT_s/dx in the beginning and at the end of the evaporation zone are assumed to be rather small $\left(\left.\frac{dT_f}{dx}\right|_{x=h_1;h_2} = \left.\frac{dT_s}{dx}\right|_{x=h_1;h_2}^{\approx 0}\right)$;

7) the heat-carrier temperature in the evaporation zone is equal to the saturation temperature and does not differ noticeably from the temperature of the particles;

8) the pressure drop is calculated from the Ergun equation [8].

Equations of the Evaporative-Cooling Model. Subject to the above-made assumptions, the equations describing the process of heat transfer in the system have the following form:

in *zone I* $(0 \le x \le h_1)$

$$J_f c_{liq} \frac{dT_f^I}{dx} = \frac{d}{dx}\left(\lambda_f^I \varepsilon \frac{dT_f^I}{dx}\right) + \frac{6(1-\varepsilon)\alpha^I}{d}\left(T_s^I - T_f^I\right), \tag{1}$$

$$0 = \frac{d}{dx}\left(\lambda_s^I (1-\varepsilon) \frac{dT_s^I}{dx}\right) + \frac{6(1-\varepsilon)\alpha^I}{d}\left(T_f^I - T_s^I\right) + Q(1-\varepsilon), \tag{2}$$

$$-\frac{dp^I}{dx} = 150 \frac{(1-\varepsilon)^2}{\varepsilon^3} \frac{\mu_{liq} u_{liq}}{d^2} + 1.75 \frac{(1-\varepsilon)}{\varepsilon^3} \frac{\rho_{liq} u_{liq}^2}{d}; \tag{3}$$

in *zone II* $(h_1 < x \le h_2)$

$$J_f \frac{di}{dx} = \frac{d}{dx}\left(\lambda_f^{II} \varepsilon \frac{dT_f^{II}}{dx}\right) + \frac{6(1-\varepsilon)\alpha^{II}}{d}\left(T_s^{II} - T_f^{II}\right), \tag{4}$$

$$0=\frac{d}{dx}\left(\lambda_{\mathrm{s}}^{\mathrm{II}}\,(1-\varepsilon)\,\frac{dT_{\mathrm{s}}^{\mathrm{II}}}{dx}\right)+\frac{6\,(1-\varepsilon)\,\alpha^{\mathrm{II}}}{d}\left(T_{\mathrm{f}}^{\mathrm{II}}-T_{\mathrm{s}}^{\mathrm{II}}\right)+Q\,(1-\varepsilon)\,, \tag{5}$$

$$-\frac{dp^{\mathrm{II}}}{dx}=150\,\frac{(1-\varepsilon)^2}{\varepsilon^3}\,\frac{\mu_{\mathrm{f}}u_{\mathrm{f}}}{d^2}+1.75\,\frac{(1-\varepsilon)}{\varepsilon^3}\,\frac{\rho_{\mathrm{f}}u_{\mathrm{f}}^2}{d}\,; \tag{6}$$

in *zone III* ($h_2<x\le h$)

$$J_{\mathrm{f}}c_{\mathrm{v}}\,\frac{dT_{\mathrm{f}}^{\mathrm{III}}}{dx}=\frac{d}{dx}\left(\lambda_{\mathrm{f}}^{\mathrm{III}}\,\varepsilon\,\frac{dT_{\mathrm{f}}^{\mathrm{III}}}{dx}\right)+\frac{6\,(1-\varepsilon)\,\alpha^{\mathrm{III}}}{d}\left(T_{\mathrm{s}}^{\mathrm{III}}-T_{\mathrm{f}}^{\mathrm{III}}\right), \tag{7}$$

$$0=\frac{d}{dx}\left(\lambda_{\mathrm{s}}^{\mathrm{III}}\,(1-\varepsilon)\,\frac{dT_{\mathrm{s}}^{\mathrm{III}}}{dx}\right)+\frac{6\,(1-\varepsilon)\,\alpha^{\mathrm{III}}}{d}\left(T_{\mathrm{f}}^{\mathrm{III}}-T_{\mathrm{s}}^{\mathrm{III}}\right)+Q\,(1-\varepsilon)\,, \tag{8}$$

$$-\frac{dp^{\mathrm{III}}}{dx}=150\,\frac{(1-\varepsilon)^2}{\varepsilon^3}\,\frac{\mu_{\mathrm{v}}u_{\mathrm{v}}}{d^2}+1.75\,\frac{(1-\varepsilon)}{\varepsilon^3}\,\frac{\rho_{\mathrm{v}}u_{\mathrm{v}}^2}{d}\,. \tag{9}$$

Boundary Conditions. With allowance for assumptions (6) and (7) and using the Danckwerts conditions [9] for the heat carrier at $x = 0$, h, we have

$$\begin{aligned}
&x=0\,,\quad c_{\mathrm{liq}}J_{\mathrm{f}}\left(T_{\mathrm{f}}^{\mathrm{I}}-T_0\right)=\lambda_{\mathrm{f}}^{\mathrm{I}}\varepsilon\,\frac{dT_{\mathrm{f}}^{\mathrm{I}}}{dx}+\lambda_{\mathrm{s}}^{\mathrm{I}}\,(1-\varepsilon)\,\frac{dT_{\mathrm{s}}^{\mathrm{I}}}{dx}\quad\text{(Danckwerts condition)}\,,\\
&\lambda_{\mathrm{s}}^{\mathrm{I}}\,(1-\varepsilon)\,\frac{dT_{\mathrm{s}}^{\mathrm{I}}}{dx}=\tilde{\alpha}\left(T_{\mathrm{s}}^{\mathrm{I}}-T_0\right)\quad\text{(preheating of heat carrier)}\,;
\end{aligned} \tag{10}$$

$$x=h_1\,,\quad p^{\mathrm{I}}=p^{\mathrm{II}}\,,\quad T_{\mathrm{f}}^{\mathrm{I}}=T_{\mathrm{f}}^{\mathrm{II}}=T_{\mathrm{sat}}\,,\quad \frac{dT_{\mathrm{f}}^{\mathrm{I}}}{dx}=\frac{dT_{\mathrm{f}}^{\mathrm{II}}}{dx}=0\,,\quad T_{\mathrm{s}}^{\mathrm{I}}=T_{\mathrm{s}}^{\mathrm{II}}\,,\quad \frac{dT_{\mathrm{s}}^{\mathrm{I}}}{dx}=\frac{dT_{\mathrm{s}}^{\mathrm{II}}}{dx}=0\,,\ \tilde{x}=0; \tag{11}$$

$$x=h_2\,,\quad p^{\mathrm{II}}=p^{\mathrm{III}}\,,\quad T_{\mathrm{f}}^{\mathrm{II}}=T_{\mathrm{f}}^{\mathrm{III}}=T_{\mathrm{sat}}\,,\quad \frac{dT_{\mathrm{f}}^{\mathrm{II}}}{dx}=\frac{dT_{\mathrm{f}}^{\mathrm{III}}}{dx}=0\,,\quad T_{\mathrm{s}}^{\mathrm{II}}=T_{\mathrm{s}}^{\mathrm{III}}\,,\quad \frac{dT_{\mathrm{s}}^{\mathrm{II}}}{dx}=\frac{dT_{\mathrm{s}}^{\mathrm{III}}}{dx}=0\,,\ \tilde{x}=1; \tag{12}$$

$$x=h\,,\quad p^{\mathrm{III}}=p_{\mathrm{atm}}\,;\quad \frac{dT_{\mathrm{f}}^{\mathrm{III}}}{dx}=0\quad\text{(Danckwerts condition)}\,,\quad \frac{dT_{\mathrm{s}}^{\mathrm{III}}}{dx}=0\,. \tag{13}$$

Integral Relations. These are the heat-balance equations derived on the basis of the heat-conduction equations (1), (2), (4), (5), (7), and (8).

Zone I. We combine Eqs. (1) and (2) and integrate the resulting equation over x from 0 to h_1:

$$\begin{aligned}
&J_{\mathrm{f}}c_{\mathrm{liq}}\left(T_{\mathrm{sat}}\,(p\,(h_1))-T_{\mathrm{f}}^{\mathrm{I}}\,(0)\right)=\\
&=\lambda_{\mathrm{f}}^{\mathrm{I}}\varepsilon\,\frac{dT_{\mathrm{f}}^{\mathrm{I}}\,(h_1)}{dx}+\lambda_{\mathrm{s}}^{\mathrm{I}}\,(1-\varepsilon)\,\frac{dT_{\mathrm{s}}^{\mathrm{I}}\,(h_1)}{dx}-\lambda_{\mathrm{f}}^{\mathrm{I}}\varepsilon\,\frac{dT_{\mathrm{f}}^{\mathrm{I}}\,(0)}{dx}-\lambda_{\mathrm{s}}^{\mathrm{I}}\,(1-\varepsilon)\,\frac{dT_{\mathrm{s}}^{\mathrm{I}}\,(0)}{dx}+Q\,(1-\varepsilon)\,h_1\,.
\end{aligned} \tag{14}$$

Using the boundary conditions (10) and (11), we obtain the sought-for heat-balance equation from Eq. (14)

$$J_f c_{liq}\left(T_{sat}(p(h_1)) - T_0\right) = Q(1-\varepsilon)\,h_1 \,, \tag{15}$$

which yields the following equation for calculating the magnitude of zone I

$$h_1 = J_f c_{liq}\left(T_{sat}(p(h_1)) - T_0\right)\big/[Q(1-\varepsilon)] \,. \tag{16}$$

Zone II. We combine Eqs. (4) and (5) and integrate over x from h_1 to h_2 subject to conditions (11) and (12):

$$J_f\,(i_v - i_{liq}) = Q(1-\varepsilon)(h_2 - h_1) \,. \tag{17}$$

Integrating the equation [10]

$$\frac{di}{dx} = L\frac{d\tilde{x}}{dx} \tag{18}$$

over x from h_1 to h_2, subject to the conditions $\tilde{x}(h_1) = 0$ and $\tilde{x}(h_2) = 1$, we write

$$i_v - i_{liq} = L \,. \tag{19}$$

For the magnitude of the evaporation zone, Eqs. (17) and (19) yield

$$\Delta h = h_2 - h_1 = \frac{J_f L}{Q(1-\varepsilon)} \,. \tag{20}$$

Zone III. An analogous operation of integration of Eqs. (7) and (8), using boundary conditions (12) and (13), gives

$$J_f c_v\left(T_f^{III}(h) - T_{sat}(p(h_2))\right) = Q(1-\varepsilon)(h - h_2) \,. \tag{21}$$

The heat-balance equation for the whole bed will be obtained as a result of summation of Eqs. (15), (17), and (21):

$$Q(1-\varepsilon)\,h = J_f\left[c_{liq}\left(T_{sat}(p(h_1)) - T_0\right) + i_v - i_{liq} + c_v\left(T_f^{III}(h) - T_{sat}(p(h_2))\right)\right]. \tag{22}$$

The physical meaning of Eq. (22) is that the heat released from the disperse bed is spent as follows: 1) to heat liquid from T_0 to $T_{sat}(p(h_1))$ in zone I; 2) to raise the enthalpy of the heat carrier from i_{liq} to i_v (liquid evaporation) in zone II; 3) to heat the vapor from $T_{sat}(p(h_2))$ up to $T_f^{III}(h)$ in zone III.

We should note that formulas (16) and (20) determine the dependence of the dimensions of zones I and II on the basic parameters of the process: J_f, Q, L, T_0, and c_{liq}.

Nondimensionalization. *Zone I* $(0 \le \xi \le \xi_1)$

$$\frac{d\theta_f^I}{d\xi} = \frac{d}{d\xi}\left(\frac{1}{\mathrm{Pe}_f^I}\frac{d\theta_f^I}{d\xi}\right) + \frac{1}{\mathrm{Pe}^I}\left(\theta_s^I - \theta_f^I\right), \tag{23}$$

$$0 = \frac{d}{d\xi}\left(\frac{1}{\mathrm{Pe}_s^I}\frac{d\theta_s^I}{d\xi}\right) + \frac{1}{\mathrm{Pe}^I}\left(\theta_f^I - \theta_s^I\right) + \hat{Q}^I \,, \tag{24}$$

$$-D_0^I\frac{d(p^I)'}{d\xi} = 150\frac{(1-\varepsilon)^2}{\varepsilon^3}\mathrm{Re}^I + 1.75\frac{(1-\varepsilon)}{\varepsilon^3}(\mathrm{Re}^I)^2 \,. \tag{25}$$

Zone II $(\xi_1 < \xi \le \xi_2)$. With allowance for $T_f^{II}(h) = T_{sat}(p^{II}(h)) \approx T_s^{II}(h)$, to describe zone II we use the equations

$$\frac{d\tilde{x}}{d\xi} = Q_L \,, \tag{26}$$

$$\frac{d\theta_f^{II}}{d\xi} = 1.405 \frac{\tilde{p}_0 - p_{atm}}{T_{sat}(p_{atm}) - T_0} \frac{1}{(p^{II})^{0.75}} \frac{d(p^{II})'}{d\xi} \,, \tag{27}$$

$$-D_0^{II} \frac{d(p^{II})'}{d\xi} = 150 \frac{(1-\varepsilon)^2}{\varepsilon^3} \mathrm{Re}^{II} + 1.75 \frac{(1-\varepsilon)}{\varepsilon^3} (\mathrm{Re}^{II})^2 \,. \tag{28}$$

Equation (26) is based on the condition that the entire heat released in zone II is spent to vaporize the liquid. Equation (27) was obtained by approximating the data of [11] on the saturation temperature for water:

$$T_{sat}(p) = 5.62 p^{0.25} + 273 \,. \tag{29}$$

Zone III $(\xi_2 < \xi \le 1)$:

$$\frac{d\theta_f^{III}}{d\xi} = \frac{d}{d\xi}\left(\frac{1}{\mathrm{Pe}_f^{III}} \frac{d\theta_f^{III}}{d\xi}\right) + \frac{1}{\mathrm{Pe}^{III}}\left(\theta_s^{III} - \theta_f^{III}\right), \tag{30}$$

$$0 = \frac{d}{d\xi}\left(\frac{1}{\mathrm{Pe}_s^{III}} \frac{d\theta_s^{III}}{d\xi}\right) + \frac{1}{\mathrm{Pe}^{III}}\left(\theta_f^{III} - \theta_s^{III}\right) + \hat{Q}^{III} \,, \tag{31}$$

$$-D_0^{III} \frac{d(p^{III})'}{d\xi} = 150 \frac{(1-\varepsilon)^2}{\varepsilon^3} \mathrm{Re}^{III} + 1.75 \frac{(1-\varepsilon)^2}{\varepsilon^3} (\mathrm{Re}^{III})^2 \,. \tag{32}$$

The boundary conditions are

$$\xi = 0 \,, \quad \theta_f^I = \frac{1}{\mathrm{Pe}_f^I} \frac{d\theta_f^I}{d\xi} + \frac{1}{\mathrm{Pe}_s^I} \frac{d\theta_s^I}{d\xi} \,, \quad \theta_s^I = 6(1-\varepsilon) \frac{\tilde{\mathrm{Pe}}}{\mathrm{Pe}_s^I} \frac{h}{d} \frac{d\theta_s^I}{d\xi} \,; \tag{33}$$

$$\xi = \xi_1 \,, \quad (p^I)' = (p^{II})' \,, \quad \theta_f^I = \theta_f^{II} = \theta_{sat} \,, \quad \frac{d\theta_f^I}{d\xi} = \frac{d\theta_s^I}{d\xi} = 0 \,, \quad \tilde{x} = 0 \,; \tag{34}$$

$$\xi = \xi_2 \,, \quad (p^{II})' = (p^{III})' \,, \quad \theta_f^{II} = \theta_f^{III} = \theta_{sat} \,, \quad \frac{d\theta_s^{III}}{d\xi} = 0 \,, \quad \tilde{x} = 1 \,; \tag{35}$$

$$\xi = 1 \,, \quad (p^{III})' = 0 \,, \quad \frac{d\theta_f^{III}}{d\xi} = \frac{d\theta_s^{III}}{d\xi} = 0 \,. \tag{36}$$

Parameters of the Evaporative-Cooling Model. *The effective thermal conductivity coefficients* [12] are

$$\lambda_f^I/\lambda_{liq}^0 = 1 + 0.03\mathrm{Re}^I\mathrm{Pr}^I , \tag{37}$$

$$\lambda_f^{III}/\lambda_v^0 = 1 + 0.03\mathrm{Re}^{III}\mathrm{Pr}^{III} , \tag{38}$$

$$\lambda_s^I/\lambda_{liq}^0 = 12 + 0.85\mathrm{Re}^I\mathrm{Pr}^I + \frac{0.3024}{(\kappa + \sigma)\,\lambda_{liq}^0}\left(\frac{T_s^I}{100}\right)^3 , \tag{39}$$

$$\lambda_s^{III}/\lambda_v^0 = 12 + 0.85\mathrm{Re}^{III}\mathrm{Pr}^{III} + \frac{0.3024}{(\kappa + \sigma)\,\lambda_v^0}\left(\frac{T_s^{III}}{100}\right)^3 . \tag{40}$$

We note that the radiative component λ_s is calculated by the method of [13].

The coefficients of interphase heat exchange [14] are

$$\mathrm{Nu}^I = \begin{cases} 0.4\left(\dfrac{\mathrm{Re}^I}{\varepsilon}\right)^{2/3}(\mathrm{Pr}^I)^{1/3} , & \mathrm{Re}^I/\varepsilon > 200 , \\ 1.6\cdot 10^{-2}\left(\dfrac{\mathrm{Re}^I}{\varepsilon}\right)^{1.3}(\mathrm{Pr}^I)^{1/3} , & \mathrm{Re}^I/\varepsilon \le 200 ; \end{cases} \tag{41}$$

$$\mathrm{Nu}^{III} = \begin{cases} 0.4\left(\dfrac{\mathrm{Re}^{III}}{\varepsilon}\right)^{2/3}(\mathrm{Pr}^{III})^{1/3} , & \mathrm{Re}^{III}/\varepsilon > 200 , \\ 1.6\cdot 10^{-2}\left(\dfrac{\mathrm{Re}^{III}}{\varepsilon}\right)^{1.3}(\mathrm{Pr}^{III})^{1/3} , & \mathrm{Re}^{III}/\varepsilon \le 200 . \end{cases} \tag{42}$$

The thermophysical characteristics of a two-phase heat carrier in zone II [1] are

$$\rho_f = \frac{\rho_v\rho_{liq}}{\rho_v + \tilde{x}\rho_{liq}} , \quad \mu_f = \mu_{liq}\,(1 - \tilde{x}) + \mu_v\tilde{x} . \tag{43}$$

Calculations by the developed model were performed for the system "water–steam" for which, based on reference values [11], the following approximating dependences were used:

$$\mu_{liq} = 0.01\left(T_f^I\right)^{-0.76} , \quad \lambda_{liq}^0 = 0.5\left(T_f^I\right)^{0.06} ; \tag{44}$$

$$\rho_v = 0.00352p^{III}/T_f^{III} , \quad \mu_v = 2.64\cdot 10^{-7}\left(T_f^{III}\right)^{0.74} , \quad \lambda_v^0 = 0.00021\left(T_f^{III}\right)^{0.84} . \tag{45}$$

Numerical Simulation of the Process of Filtrational Evaporative Cooling. The formulated boundary-value problem (23)–(32) with boundary conditions (33)–(36) was solved numerically by the method of collocations [15].

Calculation procedure for the case of existence of zone I alone. For this purpose, Eqs. (23)–(25) with conditions (33) and (36) are used.

Calculation procedure for the case of existence of zones I and II. First, using $T_{sat}(p_{atm})$ and Eq. (16), the value of $h_1^{[0]}$ was estimated; Δh was calculated from Eq. (20), and the conditions of the absence of zone III ($h_2 \geq h$) was checked. Then, within the framework of the iterative process, the following procedures were performed successively:

a) the problem for zone II was solved with the boundary conditions $\xi = \xi_1^{[k]}$, $\tilde{x} = 0$; $\xi = 1$, $\theta_f^{II} = 1$, $(p^{II})' = 0$;

b) the problem for zone I was solved with the boundary conditions (33) at $\xi = 0$, $\xi = \xi_1^{[k]}$, $(p^I)' = (p^{II})'$; $\dfrac{d\theta_f^I}{d\xi} = \dfrac{d\theta_s^I}{d\xi} = 0$;

c) the value of $h_1^{[k+1]}$ was confirmed with the aid of Eq. (16) at $T_{sat}(p(h_1^{[k]}))$, where $p(h_1^{[k]})$ is the pressure at the interface of zones I and II calculated in the course of the kth iteration ($k \geq 1$);

d) the value obtained was compared with $h_1^{[k]}$.

The condition of completion of the iteration process is

$$\left| 1 - \frac{h_1^{[k]}}{h_1^{[k+1]}} \right| \leq 10^{-3} . \tag{46}$$

Calculation procedure for the general case of the existence of all three zones. Preliminarily, the magnitude of zone I $h_1^{[0]}$ at $T_{sat}(p_{atm})$ was estimated and the condition $h_2 < h$ was checked. Thereafter, within the framework of an analogous iterative process the following procedures were carried out:

a) the problem for zone III was solved with the boundary conditions $\xi = \xi_2^{[k]}$, $\theta_f^{III} = \theta_{sat}(p(\xi_2))$, $\dfrac{d\theta_f^{III}}{d\xi} = 0$; $\xi = 1$, $(p^{III})' = 0$; $\dfrac{d\theta_f^{III}}{d\xi} = \dfrac{d\theta_s^{III}}{d\xi} = 0$;

b) the problem for zone II was solved with the boundary conditions $\xi = \xi_2^{[k]}$, $\tilde{x} = 1$, $(p^{II})' = (p^{III})'$; $\theta_f^{II} = \theta_f^{III} = \theta_{sat}(p(\xi_2))$;

c) the problem for zone I was solved with the boundary conditions (33) at $\xi = 0$, $\xi = \xi_1^{[k]}$, $(p^I)' = (p^{II})'$; $\dfrac{d\theta_f^I}{d\xi} = \dfrac{d\theta_s^I}{d\xi} = 0$;

d) the value of $h_1^{[k+1]}$ was refined by Eq. (16) at $T_{sat}(p(h_1^{[k]}))$;

e) the quantities $h_1^{[k]}$ and $h_1^{[k+1]}$ were compared;

f) the computation was considered completed if condition (46) is satisfied.

In the case where

$$h_1 = \frac{J_f c_{liq} (T_{sat}(h_1) - T_0)}{Q(1 - \varepsilon)} \geq h , \tag{47}$$

which at $T_{sat}(h_1) = T_{sat}(p_{atm})$ yields

$$\hat{Q}^I \leq 1 , \tag{48}$$

only zone I exists. For the pressure drop the following formula is obtained:

$$\frac{D^I}{D_{Er}^I} = 1.15 - 1.9 \left(\frac{d}{h}\right)^{0.25} \hat{Q}^I , \tag{49}$$

which approximates the calculated data with a standard error of about 4%.

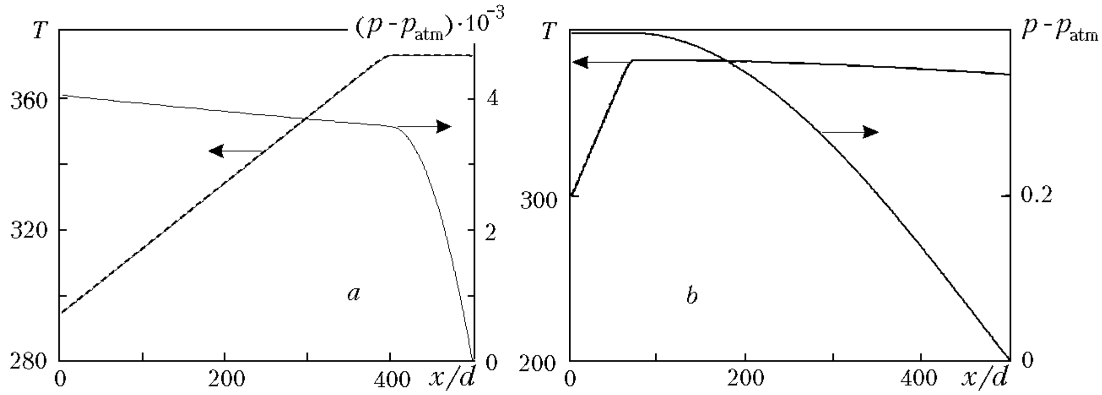

Fig. 2. Temperature and pressure profiles in a granular bed in the presence of zones I and II (h = 0.5 m, d = 0.001 m, J_f = 1 kg/(m^2·sec), T_0 = 293 K): a) $Q = 1.8\cdot10^6$ W/m^3; b) $4.3\cdot10^6$; solid lines, heat carrier; dashed lines, particles.

The results of calculations of the temperatures of phases and of the pressure drop are shown in Figs. 2 and 3; Fig. 2 presents the temperature and pressure profiles for the case of existence of zones I and II, which, with account for Eqs. (16) and (20), corresponds to the condition

$$\frac{J_f}{Q(1-\varepsilon)}\left(c_{liq}(T_{sat}(h_1)-T_0)+L\right)\geq h\,. \tag{50}$$

Relation (50) in dimensionless form, subject to $T_{sat}(p(h_1)) \approx T_{sat}(p_0)$ (see Figs. 2 and 3), takes the form

$$\frac{\theta_{sat}(p')\,|_{p'=1}}{\hat{Q}^1}+\frac{1}{Q_L}\geq 1\,. \tag{51}$$

To determine the magnitude of pressure at the inlet into the bed, the following relation was obtained:

$$\frac{D_0}{D_{Er}^{III}}=\frac{(d/h)^{-0.45}Q_L^{7.5}}{0.008-3\left(\dfrac{d}{h}\right)^{-0.38}Q_L^{6.4}}\,, \tag{52}$$

which describes the calculated values with a standard error of about 6%.

Figure 3 presents the temperature and pressure profiles for the general case where there are three zones in the heat-releasing bed, i.e.,

$$h_1+\Delta h<h\,. \tag{53}$$

Subject to (16), (20), and $T_{sat}(p(h_1)) \approx T_{sat}(p_0)$, this condition has the form

$$\frac{\theta_{sat}(p')\,|_{\xi=0}}{\hat{Q}^1}+\frac{1}{Q_L}<1\,. \tag{54}$$

The value of p_0 in this case was calculated by a formula similar to Eq. (52):

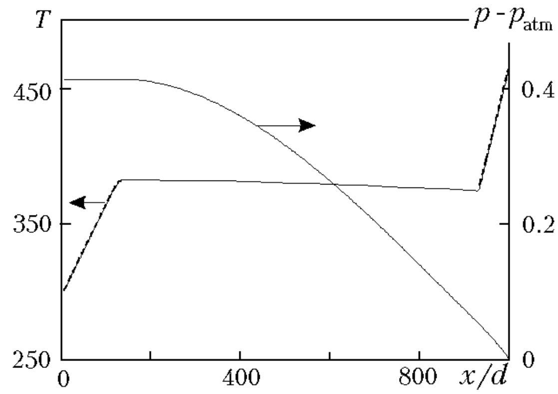

Fig. 3. Temperature and pressure profiles in a granular bed in the presence of all three zones at $Q = 4.7 \cdot 10^6$ W/m^3. The remaining parameters and symbols are same as in Fig. 2.

$$\frac{D_0}{D_{Er}^{III}} = \frac{(d/h)^{0.25} Q_L^{2.2}}{0.5 + 0.22 \left(\frac{d}{h}\right)^{0.25} Q_L^{2.2}} . \tag{55}$$

The error in using Eq. (55) does not exceed 10%.

It should be noted that, just as in the case of existence of zone I alone, so in the presence of several zones in zones I and III (Figs. 2 and 3), virtually linear profiles of T_f and T_s are realized for the given specific conditions. Here, the temperatures of water and particles differ insignificantly. Evidently, such changes in the temperatures of phases can be described by the simplified equation

$$J \frac{dT}{dx} = Q (1 - \varepsilon) . \tag{56}$$

Conclusions. Mathematical simulation of thermohydraulic processes occurring in a heat-releasing bed of spherical particles in the presence of a first-order phase transition (liquid evaporation) is carried out. For water evaporation, the profiles of temperature and pressure have been calculated for different conditions of the occurrence of the processes of heating and evaporation of a heat carrier. Based on this, a simple engineering method of calculation of the system of evaporative cooling has been developed. It includes

1) dimensionless conditions of the existence of one, two, and all three zones of cooling — Eqs. (48), (51), and (54);

2) approximating dependences for calculating the pressure drop in a granular bed — Eqs. (49), (52), and (55);

3) formulas (16) and (20) for calculating the position and magnitude of the liquid evaporation zone (on the basis of numerical calculations of the process of evaporation, the possibility of replacing $T_{sat}(p(h_1))$ in Eq. (16) by $T_{sat}(p(p_0))$, where p_0 is calculated from Eq. (52) or Eq. (55) is shown).

NOTATION

c_{liq}, c_v, specific heats of liquid and vapor at constant pressure, J/(kg·K); d, diameter of particles, m; $D_0 = \frac{p_0 - p_{atm}}{h} \frac{d^3 \tilde{\rho}_v}{(\mu_v |_{T=373K})^2}$; $D^I = \frac{p_0 - p_{atm}}{h} \frac{d^3 \rho_{liq}}{(\mu_{liq} |_{T=373K})^2}$; $D_0^I = \frac{\tilde{p}_0 - p_{atm}}{h} \frac{d^3 \rho_{liq}}{(\mu_{liq})^2}$; $D_0^{II} = \frac{\tilde{p}_0 - p_{atm}}{h} \frac{d^3 \rho_f}{(\mu_f)^2}$; $D_0^{III} = \frac{\tilde{p}_0 - p_{atm}}{h} \frac{d^3 \rho_v}{(\mu_v)^2}$; $D_{Er}^I = 150 \frac{(1-\varepsilon)^2}{\varepsilon^3} \mathrm{Re}_0^1 + 1.75 \frac{(1-\varepsilon)}{\varepsilon^3} (\mathrm{Re}_0^1)^2$; $D_{Er}^{III} = 150 \frac{(1-\varepsilon)^2}{\varepsilon^3} \mathrm{Re}_0^{III} + 1.75 \frac{(1-\varepsilon)}{\varepsilon^3} (\mathrm{Re}_0^{III})^2$; $\tilde{D}_0 =$

$150\,\dfrac{(1-\varepsilon)^2}{\varepsilon^3}\,\mathrm{Re}_0 + 1.75\,\dfrac{(1-\varepsilon)}{\varepsilon^3}\,\mathrm{Re}_0^2$; h, height of the granular bed, m; h_1, height of zone I, m; h_2, coordinate of the end of the evaporation zone, m; i, enthalpy of water-steam mixture, J/kg; i_{liq}, enthalpy of liquid saturation, J/kg; i_{v}, enthalpy of vapor saturation, J/kg; J_{f}, heat-carrier mass flow, kg/(m^2·sec); L, specific heat of vaporization, J/kg; $\mathrm{Nu}^{\mathrm{I}} = \dfrac{\alpha^{\mathrm{I}} d}{\lambda^0_{\mathrm{liq}}}$, $\mathrm{Nu}^{\mathrm{III}} = \dfrac{\alpha^{\mathrm{III}} d}{\lambda^0_{\mathrm{v}}}$, Nusselt numbers; $\mathrm{Pe}^{\mathrm{I}}_{\mathrm{f}} = \dfrac{c_{\mathrm{liq}} J_{\mathrm{f}} h}{\varepsilon \lambda^{\mathrm{I}}_{\mathrm{f}}}$, $\mathrm{Pe}^{\mathrm{III}}_{\mathrm{f}} = \dfrac{c_{\mathrm{v}} J_{\mathrm{f}} h}{\varepsilon \lambda^{\mathrm{III}}_{\mathrm{f}}}$, $\mathrm{Pe}^{\mathrm{I}} = \dfrac{c_{\mathrm{liq}} J_{\mathrm{f}} h}{6\alpha^{\mathrm{I}}(1-\varepsilon)h}$, $\mathrm{Pe}^{\mathrm{III}} = \dfrac{c_{\mathrm{v}} J_{\mathrm{f}} h}{6\alpha^{\mathrm{III}}(1-\varepsilon)h}$, $\mathrm{Pe}^{\mathrm{I}}_{\mathrm{s}} = \dfrac{c_{\mathrm{liq}} J_{\mathrm{f}} h}{(1-\varepsilon)\lambda^{\mathrm{I}}_{\mathrm{s}}}$, $\mathrm{Pe}^{\mathrm{III}}_{\mathrm{s}} = \dfrac{c_{\mathrm{v}} J_{\mathrm{f}} h}{(1-\varepsilon)\lambda^{\mathrm{III}}_{\mathrm{s}}}$, $\widetilde{\mathrm{Pe}} = \dfrac{c_{\mathrm{liq}} J_{\mathrm{f}} h}{6\tilde{\alpha}(1-\varepsilon)h}$, Peclet numbers; Pr^{I}, $\mathrm{Pr}^{\mathrm{III}}$, Prandtl numbers for liquid and vapor; p, pressure, Pa; p_0, pressure at the inlet to the granular bed, Pa; $\tilde{p}_0 = \dfrac{\mu^2_{\mathrm{liq}}(T_0)h}{\rho_{\mathrm{liq}}(T_0)d^3}\tilde{D}_0 + p_{\mathrm{atm}}$, Pa; $p' = (p - p_{\mathrm{atm}})/(\tilde{p} - p_{\mathrm{atm}})$; q_{in}, interphase heat flux, W/m^2; Q, heat release power, W/m^3; $\hat{Q}^{\mathrm{I}} = \dfrac{Q(1-\varepsilon)h}{c_{\mathrm{liq}} J_{\mathrm{f}}(T_{\mathrm{sat}}(p_{\mathrm{atm}}) - T_0)}$; $\hat{Q}^{\mathrm{III}} = \dfrac{Q(1-\varepsilon)h}{c_{\mathrm{v}} J_{\mathrm{f}}(T_{\mathrm{sat}}(p_{\mathrm{atm}}) - T_0)}$; $\hat{Q}_L = \dfrac{Q(1-\varepsilon)h}{J_{\mathrm{f}} L}$; $\mathrm{Re}^{\mathrm{I}} = \dfrac{J_{\mathrm{f}} h}{\mu_{\mathrm{liq}}}$, $\mathrm{Re}^{\mathrm{II}} = \dfrac{J_{\mathrm{f}} d}{\mu_{\mathrm{f}}}$, $\mathrm{Re}^{\mathrm{III}} = \dfrac{J_{\mathrm{f}} d}{\mu_{\mathrm{v}}}$, $\mathrm{Re}^{\mathrm{I}}_0 = \dfrac{J_{\mathrm{f}} d}{\mu_{\mathrm{liq}}\,|_{T=373\mathrm{K}}}$, $\mathrm{Re}^{\mathrm{III}}_0 = \dfrac{J_{\mathrm{f}} d}{\mu_{\mathrm{v}}\,|_{T=373\mathrm{K}}}$, $\mathrm{Re}_0 = \dfrac{J_{\mathrm{f}} d}{\mu_{\mathrm{liq}}(T_0)}$, Reynolds numbers; T, temperature, K; T_0, inlet temperature of liquid, K; T_{sat}, saturation temperature, K; u, filtration rate, m/sec; x, coordinate, m; $\tilde{x}$, mass flow rate vapor content; α, coefficient of interphase heat transfer, W/(m^2·K); $\tilde{\alpha} = c_{\mathrm{liq}} J_{\mathrm{f}} \mathrm{Re}_0^{-0.5} (\mathrm{Pr}^{\mathrm{I}})^{-0.6}$ [16], coefficient of heat transfer on the inlet surface, W/(m^2·K); ε, porosity; κ, absorption coefficient, 1/m; λ^0_{liq} and λ^0_{v}, molecular thermal conductivity of liquid and vapor, W/(m·K); λ_{f} and λ_{s}, effective thermal conductivity of a heat carrier and particles, W/(m·K); μ_{liq}, μ_{v}, μ_{f}, dynamic viscosity of liquid, vapor, and a vapor-liquid mixture, kg/(m·sec); $\theta_{\mathrm{f}} = (T_{\mathrm{f}} - T_0)/(T_{\mathrm{sat}}(p_{\mathrm{atm}}) - T_0)$; $\theta_{\mathrm{s}} = (T_{\mathrm{s}} - T_0)/(T_{\mathrm{sat}}(p_{\mathrm{atm}}) - T_0)$; $\theta_{\mathrm{sat}} = (T_{\mathrm{sat}} - T_0)/(T_{\mathrm{sat}}(p_{\mathrm{atm}}) - T_0)$; ρ_{liq}, ρ_{v}, and ρ_{f}, density of liquid, vapor, and a vapor-liquid mixture, kg/m^3; $\tilde{\rho}_{\mathrm{v}}$, density of vapor at atmospheric pressure and 373 K, kg/m^3; σ, scattering coefficient, 1/m; $\xi = x/h$; $\xi_1 = h_1/h$, $\xi_2 = h_2/h$. Superscripts: 0. molecular; I, II, III, numbers of zones. Subscripts: 0, at the inlet to the bed; atm, atmospheric; Er, by Erungen formula; f, heat carrier; in, interphase; liq, liquid; s, particles; sat, saturation; v, vapor.

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